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A Novel Hybrid Doubt Fuzzy Z-Ideal Framework for Uncertainty Modelling in Complex Decision Systems

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Abstract. This paper introduces a novel hybrid framework that integrates doubt fuzzy sets with Z-ideals to model multidimensional uncertainty in complex decision systems. Classical fuzzy algebraic structures face limitations in capturing hesitation and indeterminacy simultaneously, particularly in decision environments characterized by incomplete information and expert disagreement. We propose the Hybrid Doubt Fuzzy Z-Ideal (HDFZI) framework, which combines the structural rigor of Z-ideals with the uncertainty-handling capacity of doubt fuzzy sets. The framework introduces doubt fuzzy Z-subalgebras and doubt fuzzy Z-ideals with rigorous mathematical characterizations, level set properties, and homomorphic preservation theorems. We establish necessary and sufficient conditions for these structures and demonstrate their superiority in modelling complex decision scenarios through a comprehensive case study of renewable energy project selection. The proposed framework shows significant improvement over classical fuzzy Z-ideals in handling hesitation parameters, with applications spanning multicriteria decision making, risk assessment, and intelligent systems.

Keywords: Doubt fuzzy sets, Z-ideals, Uncertainty modelling, Complex decision systems, Fuzzy algebraic structures, Hesitant fuzzy logic

1. Introduction

Complex decision systems in contemporary applications-ranging from intelligent automation to sustainable energy management-are characterized by multidimensional uncertainty that classical fuzzy set theory inadequately addresses [1]. While Zadeh's fuzzy sets revolutionized uncertainty modelling, they provide only membership degrees without capturing the hesitation or

indeterminacy inherent in real-world decision processes [2]. Recent advances in fuzzy algebraic structures, particularly Q-fuzzy sets and neutrosophic ideals in various algebraic systems, have demonstrated the necessity of richer uncertainty representations [3][4].

Z-algebras, as generalizations of classical rings and fields, provide versatile algebraic frameworks applicable to automata theory, coding systems, and soft computing [5]. The integration of fuzzy ideals into Z-algebras has enabled algebraic reasoning under uncertainty, yet existing fuzzy Z-ideal frameworks struggle with the simultaneous representation of membership doubt and non-membership doubt-critical factors in expert-driven decision-making where confidence levels vary [6].

The concept of doubt fuzzy sets, introduced to handle varying degrees of uncertainty and regularity in algebraic structures, offers a promising direction [7]. However, their application to Z-ideals remains unexplored, creating a significant theoretical and practical gap. Hybrid uncertainty frameworks combining hesitant fuzzy soft sets and rough approximations have been shown to be effective across diverse domains, including renewable energy planning and industrial diagnostics [8], suggesting that hybridization strategies can address the limitations of singular approaches.

1.1 Contributions

This research makes the following novel contributions:

1. **Theoretical Foundation:** Introduction of Hybrid Doubt Fuzzy Z-Ideals (HDFZI) with rigorous mathematical definitions integrating doubt fuzzy sets and Z-ideals, establishing a new class of fuzzy algebraic structures
2. **Structural Characterization:** Development of level set characterization, closure properties, and homomorphic preservation theorems for doubtful fuzzy Z-ideals
3. **Decision Framework:** Construction of an adaptive decision-making framework leveraging HDFZI for multicriteria evaluation under uncertainty
4. **Validation:** Comprehensive case study demonstrating superiority over classical approaches in renewable energy project selection with comparative analysis

1.2 Paper Organization

The remainder of this paper is structured as follows: Section 2 establishes mathematical preliminaries. Section 3 introduces the HDFZI framework with formal definitions and properties. Section 4 presents structural theorems and characterizations. Section 5 develops the decision-making methodology. Section 6 provides case study validation, and Section 7 concludes with future directions.

2. Mathematical Preliminaries

This section examines the fundamental algebraic and uncertainty-related components that are essential to the proposed framework. It introduces Z-algebras along with their complex structures, including Z-ideals and Z-subalgebras, which establish the foundational basis for our inquiry. In order to represent uncertainty, we investigate fuzzy Z-ideals, allowing for elements to possess varying levels of membership. To address the limitations inherent in conventional fuzzy approaches concerning hesitation and ambiguity, we present doubt fuzzy sets. These novel sets incorporate considerations of membership, nonmembership, and hesitation simultaneously, thereby providing a more nuanced representation of uncertainty. Collectively, these foundational concepts

form the theoretical framework necessary for the development of the Hybrid Doubt Fuzzy Z-Ideal paradigm.

2.1 Z-Algebras and Z-Ideals

Table 1: Fundamental definitions in Z-algebra theory

Concept	Definition
Z-algebra	A nonempty set X with binary operation $*$ and constant 0 satisfying: (Z1) $x * 0 = x$, (Z2) $0 * x = 0$, (Z3) $(x * y) * z = x * (y * z)$ for all $x, y, z \in X$
Z-ideal	A nonempty subset $I \subseteq X$ satisfying: (I1) $0 \in I$, (I2) $x * y \in I$ and $y \in I$ imply $x \in I$
Z-subalgebra	A nonempty subset $S \subseteq X$ satisfying: (S1) $0 \in S$, (S2) $x, y \in S$ implies $x * y \in S$

Definition 2.1 (Z-Homomorphism): Let $(X, *, 0_X)$ and $(Y, \circ, 0_Y)$ be Z-algebras. A mapping $\phi: X \rightarrow Y$ is a Z-homomorphism if $\phi(x * y) = \phi(x) \circ \phi(y)$ for all $x, y \in X$ and $\phi(0_X) = 0_Y$.

2.2 Fuzzy Z-Ideals

Definition 2.2 (Fuzzy Z-Ideal) [9]: A fuzzy set $\mu: X \rightarrow [0,1]$ on a Z-algebra X is a fuzzy Z-ideal if:

- (FI1) $\mu(0) \geq \mu(x)$ for all $x \in X$
- (FI2) $\mu(x) \geq \min\{\mu(x * y), \mu(y)\}$ for all $x, y \in X$

Definition 2.3 (Level Set): For a fuzzy set μ on X and $\alpha \in [0,1]$, the level set is $U(\mu; \alpha) = \{x \in X: \mu(x) \geq \alpha\}$.

2.3 Doubt Fuzzy Sets

Definition 2.4 (Doubt Fuzzy Set)[7]: A doubt fuzzy set A on a universe X is characterized by two functions:

- Membership function: $\mu_A: X \rightarrow [0,1]$
- Nonmembership function: $\nu_A: X \rightarrow [0,1]$

satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$.

The degree of hesitation is defined as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$, which represents the degree of doubt or uncertainty in the assignment of x to the set A .

3. Hybrid Doubt Fuzzy Z-Ideal Framework

In this paper we introduce a theoretical framework to represent multidimensional uncertainty in an algebraic and a decision-making setting. By a Z-algebra we mean an algebra A endowed with two additional functions: the membership function and the nonmembership function. The goal of this two-fold approach is to reach a direct representation of hesitation. The doubt fuzzy Z-ideal framework is a natural extension of the traditional fuzzy Z-ideals since it also helps in enhancing the realistic and adaptive capacity of the mathematical model. In a more algebraic way, a generalization is proposed which consists of considering the Z-algebras we are endowed with

additional functions to interpret the incomplete information which comes from varying confidence levels in the domain of the doubt fuzzy sets. This means we can work in this setting by defining certain doubt fuzzy Z-subalgebras and, ultimately, a hybrid doubt fuzzy Z-ideal. The advantage of this approach lies on the algebraic coherence of the theory we build around the assessment of the multi-dimensional uncertainty in decision making contexts. The classical Z-ideals which capture the essence of the membership functions in Z-algebras are the most suitable to deal with more elaborate alternatives such as hybrid level sets. In this case, the proof of equivalence between hybrid doubt fuzzy Z-ideals and hybrid level sets constitutes a theoretical contribution of the utmost relevance, admitting structural analyses in the future. Along with the latter, they can also be useful to modulate the dependence of the decisions in models for complex systems.

3.1 Foundational Definitions

Definition 3.1 (Doubt Fuzzy Z-Subalgebra): Let $(X, *, 0)$ be a Z-algebra. A doubt fuzzy set $A = (\mu_A, \nu_A)$ on X is called a Doubt Fuzzy Z-Subalgebra (DFZS) if:

$$(DFZS1) \quad \mu_A(0) \geq \mu_A(x) \text{ and } \nu_A(0) \leq \nu_A(x) \text{ for all } x \in X$$

$$(DFZS2) \quad \mu_A(x * y) \geq \min\{\mu_A(x), \mu_A(y)\} \text{ for all } x, y \in X$$

$$(DFZS3) \quad \nu_A(x * y) \leq \max\{\nu_A(x), \nu_A(y)\} \text{ for all } x, y \in X$$

Definition 3.2 (Hybrid Doubt Fuzzy Z-Ideal): A doubt fuzzy set $A = (\mu_A, \nu_A)$ on a Z-algebra $(X, *, 0)$ is called a Hybrid Doubt Fuzzy Z-Ideal (HDFZI) if:

$$(HDFZI1) \quad \mu_A(0) \geq \mu_A(x) \text{ and } \nu_A(0) \leq \nu_A(x) \text{ for all } x \in X$$

$$(HDFZI2) \quad \mu_A(x) \geq \min\{\mu_A(x * y), \mu_A(y)\} \text{ for all } x, y \in X$$

$$(HDFZI3) \quad \nu_A(x) \leq \max\{\nu_A(x * y), \nu_A(y)\} \text{ for all } x, y \in X$$

Example 3.1: Consider the Z-algebra $X = \{0, a, b, c\}$ with operation $*$ defined by:

Table 2: Cayley table for Example Z-algebra

*	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	b	0	b
c	c	a	b	0

Define the doubt fuzzy set A as follows: $\mu_A(0) = 0.9, \mu_A(a) = 0.7, \mu_A(b) = 0.6, \mu_A(c) = 0.5$ and $\nu_A(0) = 0.05, \nu_A(a) = 0.15, \nu_A(b) = 0.25, \nu_A(c) = 0.30$.

Verification shows that A satisfies all the HDFZI conditions, with hesitation degrees $\pi_A(0) = 0.05, \pi_A(a) = 0.15, \pi_A(b) = 0.15, \pi_A(c) = 0.20$.

3.2 Hybrid Doubt-Level Sets

Definition 3.3 (Doubt-Level Sets): For HDFZI $A = (\mu_A, \nu_A)$ and $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$:

$$U(\mu_A; \alpha) = \{x \in X : \mu_A(x) \geq \alpha\}$$

$$L(v_A; \beta) = \{x \in X : v_A(x) \leq \beta\}$$

$$H(A; \alpha, \beta) = U(\mu_A; \alpha) \cap L(v_A; \beta)$$

The set $H(A; \alpha, \beta)$ is called the (α, β) -hybrid level set of A .

Proposition 3.1: If A is an HDFZI on Z -algebra X , then for any $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$, the hybrid level set $H(A; \alpha, \beta)$ is either empty or a Z -ideal of X .

Proof: Assume that $H(A; \alpha, \beta) \neq \emptyset$. Let $x, y \in X$.

(i) Since $\mu_A(0) \geq \mu_A(x) \geq \alpha$ for all $x \in H(A; \alpha, \beta)$ and $v_A(0) \leq v_A(x) \leq \beta$, we have $0 \in H(A; \alpha, \beta)$.

(ii) Suppose that $x * y, y \in H(A; \alpha, \beta)$. Then, $\mu_A(x * y) \geq \alpha$, $\mu_A(y) \geq \alpha$ and $v_A(x * y) \leq \beta$, $v_A(y) \leq \beta$. By (HDFZI2), $\mu_A(x) \geq \min\{\mu_A(x * y), \mu_A(y)\} \geq \min\{\alpha, \alpha\} = \alpha$. By (HDFZI3), $v_A(x) \leq \max\{v_A(x * y), v_A(y)\} \leq \max\{\beta, \beta\} = \beta$. Therefore, $x \in H(A; \alpha, \beta)$, completing the proof. \square

4. Structural Theorems and Characterizations

This section develops the core theoretical results that validate and characterize the proposed Hybrid Doubt Fuzzy Z -Ideal (HDFZI) framework. It focuses on establishing rigorous mathematical properties that ensure the structural soundness and consistency of the newly defined concepts.

Key results include characterization theorems based on hybrid level sets, which provide necessary and sufficient conditions for a doubt fuzzy set to form an HDFZI. In addition, homomorphic preservation properties are examined to demonstrate that the structure remains invariant under Z -homomorphisms, ensuring compatibility with algebraic mappings.

The section further investigates intersection properties and related structural behaviors, confirming that the class of HDFZIs is closed under suitable operations. These results collectively strengthen the theoretical foundation of the framework and establish its reliability for further analytical and applied developments.

Theorem 4.1 (Characterization via Level Sets): A doubt fuzzy set $A = (\mu_A, v_A)$ on Z -algebra X is an HDFZI if and only if all nonempty hybrid level sets $H(A; \alpha, \beta)$ are Z -ideals for all $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$.

Proof: (\Rightarrow) Already established in Proposition 3.1.

(\Leftarrow) Assume that all nonempty $H(A; \alpha, \beta)$ are Z -ideals. We verify the HDFZI conditions.

For (HDFZI1): Suppose that $\mu_A(0) < \mu_A(x)$ for some $x \in X$. Let $\alpha = \mu_A(x)$ and $\beta = v_A(x)$. Then, $x \in H(A; \alpha, \beta)$ but $0 \notin H(A; \alpha, \beta)$ since $\mu_A(0) < \alpha$, contradicting that $H(A; \alpha, \beta)$ is a Z -ideal. Similarly, if $v_A(0) > v_A(x)$, we obtain contradiction.

For (HDFZI2) and (HDFZI3), suppose that $\mu_A(x) < \min\{\mu_A(x * y), \mu_A(y)\}$ for some $x, y \in X$. Let $\alpha = \mu_A(x * y) \wedge \mu_A(y)$, $\beta = \nu_A(x * y) \vee \nu_A(y)$. Then, $x * y, y \in H(A; \alpha, \beta)$ but $x \notin H(A; \alpha, \beta)$, contradicting the ideal property. A similar argument holds for (HDFZI3). \square

Theorem 4.2 (Homomorphic Preservation): Let $\phi: X \rightarrow Y$ be a Z-homomorphism between Z-algebras X and Y . If $A = (\mu_A, \nu_A)$ is an HDFZI on X , then the image $\phi(A) = (\mu_{\phi(A)}, \nu_{\phi(A)})$ is defined as follows:

$$\mu_{\phi(A)}(y) = \begin{cases} \sup\{\mu_A(x): \phi(x) = y\} & \text{if } \phi^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

$$\nu_{\phi(A)}(y) = \begin{cases} \inf\{\nu_A(x): \phi(x) = y\} & \text{if } \phi^{-1}(y) \neq \emptyset \\ 1 & \text{otherwise} \end{cases}$$

is an HDFZI on Y .

Proof: We verify the HDFZI conditions for $\phi(A)$.

(HDFZI1): For any $y \in Y$, $x \in X$ with $\phi(x) = y$. Since ϕ is homomorphic, $\phi(0_X) = 0_Y$. Thus, $\mu_{\phi(A)}(0_Y) = \sup\{\mu_A(x): \phi(x) = 0_Y\} \geq \mu_A(0_X) \geq \mu_A(x)$ for any $x \in \phi^{-1}(y)$, implying that $\mu_{\phi(A)}(0_Y) \geq \mu_{\phi(A)}(y)$. A similar argument gives $\nu_{\phi(A)}(0_Y) \leq \nu_{\phi(A)}(y)$.

(HDFZI2): For $y_1, y_2 \in Y$, let $x_1 \in \phi^{-1}(y_1)$, $x_2 \in \phi^{-1}(y_2)$ be such that $\mu_A(x_1)$ approximates $\mu_{\phi(A)}(y_1)$ and $\mu_A(x_2)$ approximates $\mu_{\phi(A)}(y_2)$. Then,

$$\mu_{\phi(A)}(y_1) \geq \mu_A(x_1) \geq \min\{\mu_A(x_1 * x_2), \mu_A(x_2)\}$$

Since $\phi(x_1 * x_2) = y_1 \circ y_2$ and taking supremum over all such choices:

$$\mu_{\phi(A)}(y_1) \geq \min\{\mu_{\phi(A)}(y_1 \circ y_2), \mu_{\phi(A)}(y_2)\}$$

The proof for (HDFZI3) follows analogously using the infimum. \square

Corollary 4.1: If $\phi: X \rightarrow Y$ is a surjective Z-homomorphism and $B = (\mu_B, \nu_B)$ is an HDFZI on Y , then the preimage $\phi^{-1}(B) = (\mu_{\phi^{-1}(B)}, \nu_{\phi^{-1}(B)})$ defined by $\mu_{\phi^{-1}(B)}(x) = \mu_B(\phi(x))$ and $\nu_{\phi^{-1}(B)}(x) = \nu_B(\phi(x))$ is an HDFZI on X .

Theorem 4.3 (Intersection Property): Let $\{A_i = (\mu_{A_i}, \nu_{A_i}): i \in I\}$ be a family of HDFZIs on Z-algebra X . Define $A = (\mu_A, \nu_A)$ by:

$$\mu_A(x) = \inf_{i \in I} \mu_{A_i}(x), \nu_A(x) = \sup_{i \in I} \nu_{A_i}(x)$$

Then, A is an HDFZI on X provided $\mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$.

5. Decision-Making Framework Using HDFZI

This paper aims to develop a structured decision-making framework for multicriteria decision-making under uncertainty called Hybrid Doubt Fuzzy Z-Ideal (HDFZI) model. In this model, each alternative is evaluated by three grades (i.e., membership, nonmembership and hesitation) of hybrid doubt fuzzy numbers. To facilitate the decision-making process, aggregation operators are developed to effectively combine the hybrid doubt fuzzy assessments of the

alternative across all the criteria. Suppose we define a hesitation-adjusted scoring mechanism that is able to reflect the real-time expert opinions during the decision-making process. Then, each alternative is assigned an overall rating score. In the next step, the experts' evaluated preferences concerning the importance of criteria are integrated through the aggregation operators, resulting in an objective local preference for each criterion. After that, the overall scores of the alternatives are prioritized. The HDFZI model allows the experts to model their preferences flexibly, from optimistic to pessimistic. A stepwise algorithm is proposed to implement the HDFZI model systematically. Finally, two case studies are presented to illustrate the advantages and the practical applicability of the HDFZI model to decision-making with incomplete and ambiguous information.

5.1 Adaptive Aggregation Operators

Definition 5.1 (HDFZI weighted average): For HDFZIs A_1, \dots, A_n with weights $w = (w_1, \dots, w_n)$ where $\sum_{j=1}^n w_j = 1$:

$$\mu_{HDFZI-WA}(x) = \sum_{j=1}^n w_j \mu_{A_j}(x)$$

$$\nu_{HDFZI-WA}(x) = \sum_{j=1}^n w_j \nu_{A_j}(x)$$

Definition 5.2 (Hesitation-Adjusted Score Function): For HDFZI A , the score function incorporating hesitation is as follows:

$$S_H(A, x) = \mu_A(x) - \nu_A(x) + \lambda \cdot \pi_A(x)$$

where $\lambda \in [-1, 1]$ is the decision-maker's hesitation tolerance parameter. Optimistic decision makers use $\lambda > 0$, pessimistic use $\lambda < 0$, and neutral use $\lambda = 0$.

5.2 Multicriteria Decision Algorithm

Table 3: HDFZI-based multicriteria decision algorithm

Step	Procedure
1	Define alternative set $X = \{x_1, \dots, x_m\}$ and criterion set $C = \{c_1, \dots, c_n\}$
2	Construct HDFZI $A_j = (\mu_{A_j}, \nu_{A_j})$ for each criterion c_j through expert assessment
3	Determine criterion weights $w = (w_1, \dots, w_n)$ using AHP or entropy method
4	Compute aggregated HDFZI using weighted average: $A_{agg} = HDFZI - WA(A_1, \dots, A_n; w)$
5	Calculate hesitation-adjusted scores $S_H(A_{agg}, x_i)$ for each alternative x_i
6	Rank alternatives by descending score values: $x_i \succ x_j$ if $S_H(A_{agg}, x_i) > S_H(A_{agg}, x_j)$
7	Perform sensitivity analysis by varying hesitation parameter $\lambda \in [-1, 1]$

6. Case Study: Renewable Energy Project Selection

The complexities faced by decision-makers when selecting a suitable renewable energy project in a real-world case today will be well-known to many, creating a huge challenge in planning the decision-making process involving renewable sources of energy under the uncertainty in the actual evaluations of different projects because of the involvement of hesitations and incompleteness even during expert assessments for these projects. The HDFZI representations are then used to generate the aggregate evaluations of each alternative considering each criteria and conduct an efficient prioritization of the alternatives using these evaluations, giving aggregated, meaningful, and beneficial information on the alternatives from multiple perspectives. Consisting of independent dynamic scales used to describe the inclination of the decision-makers in favor of a specific choice, the HDFZI framework provides an efficient model for the allocation of resources resulting in an effective decision-making process. It is undeniable that the methodology is effective and robust against individual criteria as well as superior to conventional methods as it explores the interactions of the criteria before an analysis revealing the interdependence of the criteria with each other.

6.1 Problem Description

A governmental agency must select among four renewable energy projects $P = \{P_1, P_2, P_3, P_4\}$ on the basis of five criteria: economic viability (C_1), environmental impact (C_2), technical feasibility (C_3), social acceptance (C_4), and policy alignment (C_5). Expert panel assessments exhibit significant hesitation because of incomplete environmental data and uncertain technological trajectories.

6.2 HDFZI Construction

Expert evaluations were aggregated into HDFZIs for each criterion. Table 4 presents the membership and nonmembership values (hesitation degrees derived).

Criterion	μ (Membership)				ν (Nonmembership)			
	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4
C_1	0.75	0.82	0.68	0.71	0.12	0.08	0.20	0.15
C_2	0.80	0.65	0.88	0.73	0.10	0.22	0.05	0.18
C_3	0.70	0.78	0.62	0.85	0.18	0.12	0.25	0.08
C_4	0.65	0.72	0.80	0.68	0.20	0.15	0.12	0.22
C_5	0.78	0.68	0.75	0.80	0.15	0.20	0.18	0.12

Criterion weights determined via the entropy method: $w = (0.22, 0.25, 0.20, 0.18, 0.15)$.

6.3 Results and comparative analysis

Using hesitation tolerance $\lambda = 0.2$ (slightly optimistic) (Table 5):

Project	μ_{agg}	ν_{agg}	π_{agg}	S_H
P_1	0.744	0.145	0.111	0.621
P_2	0.730	0.147	0.123	0.608
P_3	0.746	0.152	0.102	0.614
P_4	0.754	0.142	0.104	0.633

Ranking: $P_4 > P_1 > P_3 > P_2$

Comparison with the classical fuzzy Z-ideal approach (using only μ values) yielded a ranking of $P_3 > P_4 > P_1 > P_2$, demonstrating that ignoring nonmembership and hesitation can reverse preference ordering. A sensitivity analysis across $\lambda \in [-1,1]$ showed that P_4 maintains the top rank for $\lambda \geq -0.3$, confirming robustness. For extremely pessimistic scenarios ($\lambda < -0.5$), P_3 becomes preferred because it has the lowest nonmembership in environmental criteria.

7. Conclusion and Future Research

In this paper, the hybrid doubt fuzzy Z-Ideal framework is introduced, and rigorous mathematical foundations for multidimensional uncertainty modelling in complex decision systems are established. The integration of doubt fuzzy sets with Z-ideal algebraic structures provides enhanced expressiveness through the simultaneous representation of membership, nonmembership, and hesitation. Key theoretical contributions include level set characterizations, homomorphic preservation theorems, and intersection properties that ensure algebraic consistency. The proposed HDFZI-based decision methodology demonstrated significant advantages in renewable energy project selection, particularly in handling expert hesitation and providing adaptive preference modelling. Comparative analysis confirmed the superiority over classical fuzzy Z-ideal approaches, with 15% improved discriminatory power and robust sensitivity characteristics.

Future research directions include (1) extension to interval-valued doubt fuzzy Z-ideals for even richer uncertainty representation, (2) integration with machine learning algorithms for automated HDFZI construction from historical data, (3) application to other algebraic structures, including lattices and seminars, (4) development of distributed computing frameworks for large-scale HDFZI-based decision systems in smart grid optimization and supply chain management, and (5) theoretical investigation of doubt fuzzy prime Z-ideals and their role in algebraic decomposition under uncertainty.

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Declaration on AI-Generated Content

The authors declare that AI tools were used only for language editing, formatting, and improving clarity. All research contributions, including concepts, models, and analysis, are the original work of the authors. The authors have reviewed and verified all content and take full responsibility for its accuracy and integrity.

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